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Measurement of elastic shear strain by white-beam SR topography

By Jaroslav Brádler[1], Milena Polcarová[1], Juliana Gemperlová[1], Amand George[2] and Alain Jacques[2]

[1] *Institute of Physics AS CR, Na Slovance 2, 182 21 Prague 8, Czech Republic*
[2] *Laboratoire de Physique des Matériaux, Ecole des Mines, Parc de Saurupt, 54042 Nancy, France*

An external tensile stress applied to an elastically anisotropic cubic material along a direction different to that of the cubic axes can cause a shear deformation. In bicrystals the shear strain can result in a mutual tilt of some planes at the grain boundary. Taking advantage of the high sensitivity of the white-beam synchrotron radiation topography to the tilt of diffracting planes, the small elastic shear strain was measured using suitable bicrystals.

Keywords: small elastic strain; synchrotron radiation topography

1. Introduction

It follows from the theory of elasticity that non-zero shear-stress components can exist in a uniaxially loaded body if the material is elastically anisotropic. The corresponding strain is very small for slightly anisotropic materials and hardly measurable, even by very sensitive methods such as double-crystal X-ray diffractometry. Investigation of bicrystals offers an interesting new possibility: uniaxial loading of some symmetric bicrystals along a certain axis can result in shear deformation which has opposite sign in the two grains. Then a tilt of certain crystallographic planes occurs which is measurable by sufficiently sensitive X-ray methods.

In this paper, results of an X-ray topography observation of elastic shear deformation in an Fe-4at%Si bicrystal are presented.

2. Theoretical

In distorted elastic materials where rotations occur bodily, only the effect of a displacement function $\boldsymbol{u}$ is considered. When the displacements are small, and we confine ourselves to the classical theory of elasticity of small deformations, the strain tensor is defined as

$$\varepsilon_{ij} = \frac{1}{2}\left[\left(\frac{\partial u_i}{\partial x_j}\right) + \left(\frac{\partial u_j}{\partial x_i}\right)\right].$$

It is symmetric: $\varepsilon_{ij} = \varepsilon_{ji}$. A generalized Hooke's law relates the strain and stresses: $\sigma_{ij} = c_{ijkl}\varepsilon_{kl}$, $\varepsilon_{ij} = s_{ijkl}\sigma_{kl}$. The fourth rank tensors c_{ijkl} and s_{ijkl} are called the elastic stiffness tensor and compliance tensor, respectively. Generally, they have 21

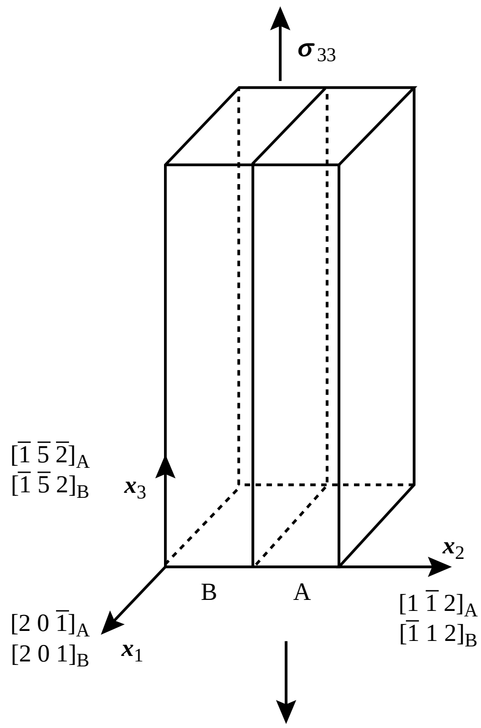

Figure 1. Coordinate system connected with the bicrystal specimen. The intersection line of the incidence plane ($x_3 = 0$) and the specimen surface is parallel to the $[1\bar{1}2]_A = [\bar{1}12]_B$ direction.

independent components as both ε_{ij} and σ_{ij} are symmetric. Being tensors they obey the normal transformation rule $c'_{ijkl} = a_{ip}a_{jq}a_{kr}a_{ls}c_{pqrs}$, where a_{ip} are direction cosines between two sets of orthonormal coordinates (x_j and x'_j). For cubic crystals the s'_{ijkl} (and similarly c'_{ijkl}) can be expressed as

$$s'_{ijkl} = s_{12}\delta_{ij}\delta_{kl} + \tfrac{1}{4}s_{44}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + \Theta\sum_{r=1}^{3} a_{ir}a_{jr}a_{kr}a_{lr},$$

where $\Theta = s_{11} - s_{12} - \frac{1}{2}s_{44}$, s_{11}, s_{12} and s_{44} are invariants, δ_{ij} is Kronecker's delta and a_{ij} are direction cosines between the basic system [100], [010] and [001] and the general one. Note that $\Theta = 0$ for isotropic crystals.

The studied symmetrical bicrystal has been loaded uniaxially along the x_3 direction in a sample coordinate system, i.e. only $\sigma_{33} = \sigma$ is different from zero and then $\varepsilon_{ij} = s_{ij33}$. The sample coordinate systems for grains A and B are presented in table 1 and figure 1. From table 1 it can be seen that only the compliances s_{1233} and s_{2333} have different signs in grains A and B; all other components have the same values in both grains. Therefore $\varepsilon^{A}_{12} = -\varepsilon^{B}_{12}$ and $\varepsilon^{A}_{23} = -\varepsilon^{B}_{23}$. For all other components the equality is valid in both grains and the bicrystal is compatible ($\varepsilon^{A}_{11} = \varepsilon^{B}_{11}$, $\varepsilon^{A}_{33} = \varepsilon^{B}_{33}$, $\varepsilon^{A}_{13} = \varepsilon^{B}_{13}$), i.e. no additional elastic stresses due to incompatibility appear (Gemperlová *et al.* 1989).

Table 1. *Direction cosines between the sample coordinate system and the basic cubic system in both grains*

grain A	100	010	001	grain B	100	010	001
$20\bar{1}$	$2/\sqrt{5}$	0	$-1/\sqrt{5}$	201	$2/\sqrt{5}$	0	$1/\sqrt{5}$
$\bar{1}1\bar{2}$	$-1/\sqrt{6}$	$1/\sqrt{6}$	$-2/\sqrt{6}$	$1\bar{1}\bar{2}$	$1/\sqrt{6}$	$-1/\sqrt{6}$	$-2/\sqrt{6}$
152	$1/\sqrt{30}$	$5/\sqrt{30}$	$2/\sqrt{30}$	$15\bar{2}$	$1/\sqrt{30}$	$5/\sqrt{30}$	$-2/\sqrt{30}$

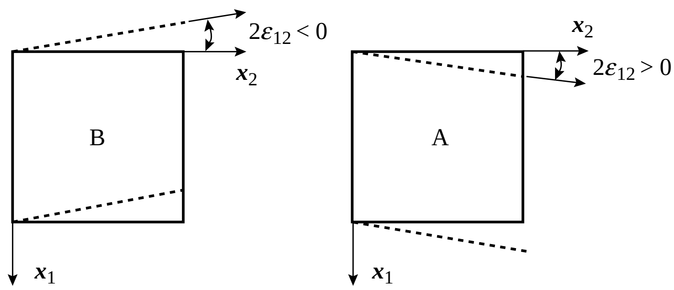

Figure 2. Schematic drawing of the shear deformation in the grains A and B of the bicrystal. The tensile stress σ_{33} is normal to the shear plane.

The shear components of the strain in the grains A and B,

$$\varepsilon_{12}^{\mathrm{A}} = \frac{1}{5\sqrt{30}}\Theta\sigma_{33} = -\varepsilon_{12}^{\mathrm{B}},$$

result in a relative tilt of planes Δ perpendicular to the x_1-axis (figure 2):

$$\Delta = 2\varepsilon_{12}^{\mathrm{B}} - 2\varepsilon_{12}^{\mathrm{A}} = -\frac{4}{5\sqrt{30}}\Theta\sigma_{33}.$$

Δ depends on the applied stress and on the constant Θ. For Fe-4at%Si, $\Theta = 7.99 \times 10^{-12}\ \mathrm{Pa}^{-1}$.

3. Experimental

For experiments a $\Sigma3$ bicrystal (characterized by a rotation of 70.5° around the [110] direction) with symmetrical grain boundary (GB) parallel to the $\{112\}_{\mathrm{A,B}}$ plane was chosen. Specimens of special shape were cut to fit in the deformation stage constructed for X-ray topography investigation under applied stress. The active part of the specimen was 10 mm long, 1.4 mm wide and 0.5 mm thick. Back-reflection topographs were taken using white-beam synchrotron radiation (SR) at LURE (Orsay, France). Symmetric reflection $40\bar{2}_{\mathrm{A}} = 402_{\mathrm{B}}$ common for both grains with $\theta = 65°$ ($\lambda = 0.116$ nm) was used. The diffracted beam was observed by an X-ray sensitive camera and recorded on Ilford L4 nuclear plates (for experimental details, see Polcarová *et al*. (1998)). The specimens were deformed by tension and topographs were taken at different stages of deformation.

The first topograph was taken at low tensile stress $\sigma_{33} = 23$ MPa that was just high enough to keep the specimen in position (figure 3*a*). The slight overlap of the images of the two grains is due to a small misorientation additional to that describing the $\Sigma3$

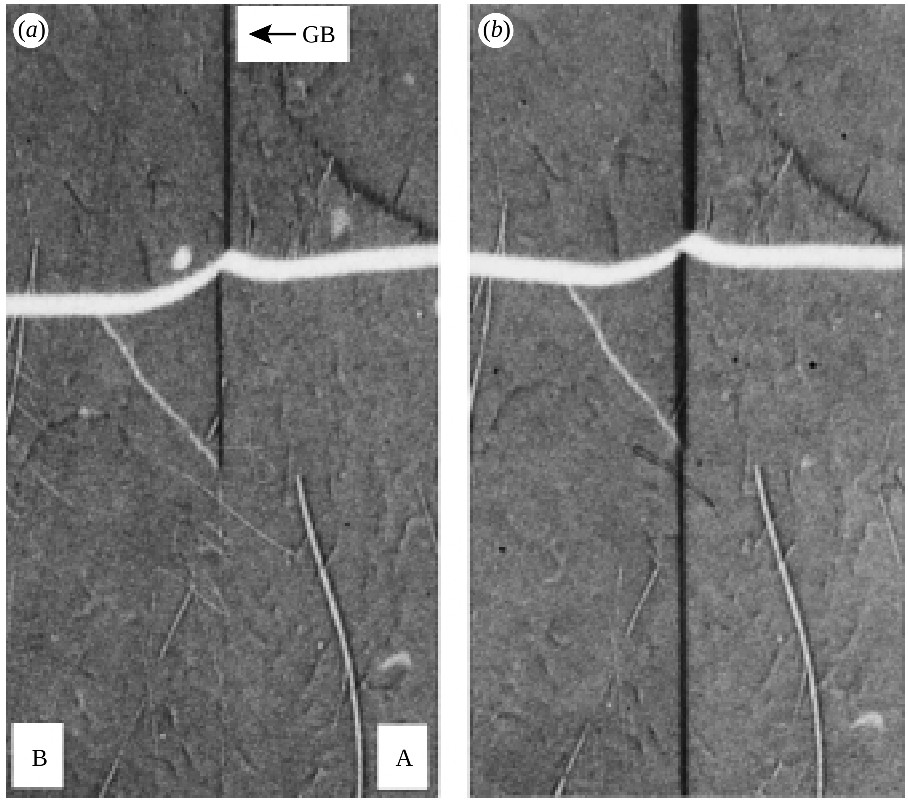

Figure 3. Topographs taken when increasing the tensile stress.
Stress values: (*a*) $\sigma = 23$ MPa; (*b*) $\sigma = 163$ MPa.

bicrystal. In the topograph taken after the stress increased to 163 MPa this overlap is larger (figure 3*b*). That means that the tilt of the diffracting planes increased. The difference of the tilts calculated from the measured overlaps is $\Delta_{\text{meas}} = -21''$. The theoretical value corresponding to the stress difference is $\Delta_{\text{theor}} = -34''$.

Figure 4 shows topographs taken at stresses 187 and 16 MPa. In the last topograph the overlap decreased again giving evidence that the tilt was due to an elastic strain. The difference is $\Delta_{\text{meas}} = 28''$, while the corresponding theoretical difference is $\Delta_{\text{theor}} = 41''$. Regarding the estimated precision of the tilt measurement, $8''$, the agreement is satisfactory.

4. Conclusions

(i) In an elastically anisotropic crystal, a shear-strain component appears when either tensile or compression stress is applied.

(ii) In slightly anisotropic crystals the shear strain is too small (*ca.* 10^{-5}) to be measured in single crystals.

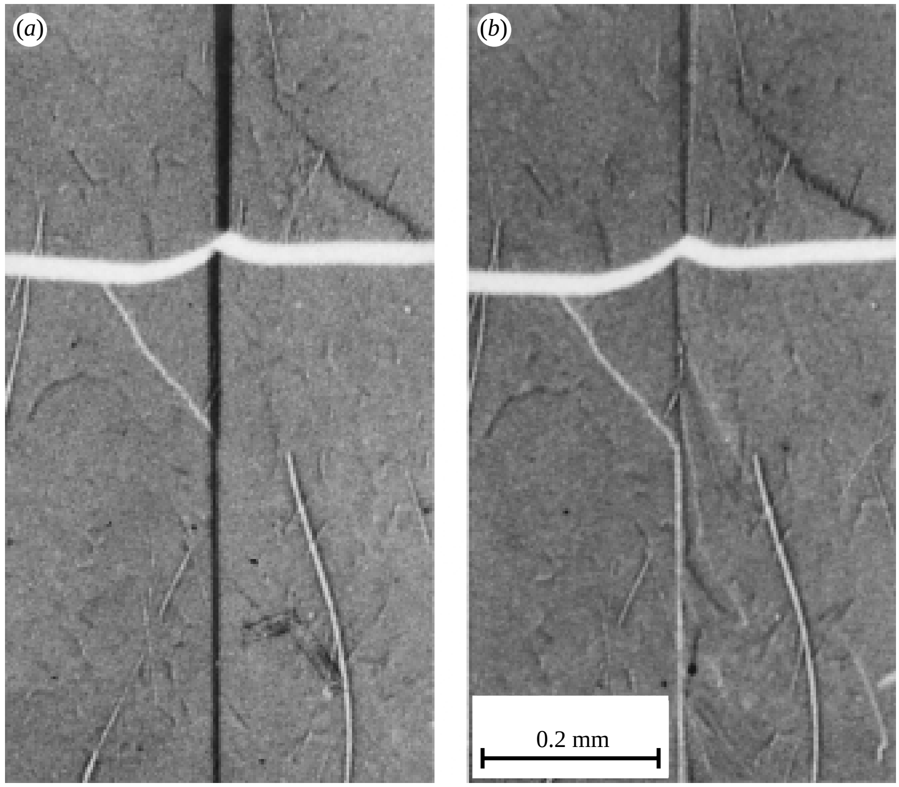

Figure 4. Topographs taken when decreasing the stress.
Stress values: (*a*) $\sigma = 187$ MPa; (*b*) $\sigma = 16$ MPa.

(iii) In special bicrystals the shear components can have opposite signs in the two grains and result in a relative tilt of order of tens of arc seconds on some sets of crystallographic planes.

(iv) This tilt causes a slight gap or overlap of X-ray topographic images of the grains.

(v) Using highly sensitive white-beam SR topography, the mutual shift of the images can be measured in dependence on applied stress (*in situ*).

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